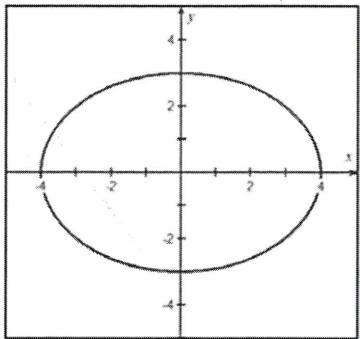
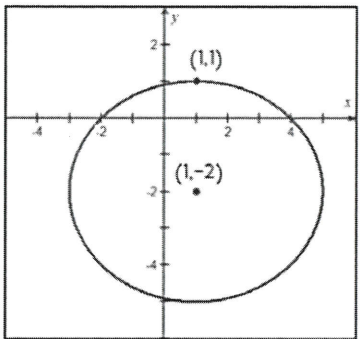
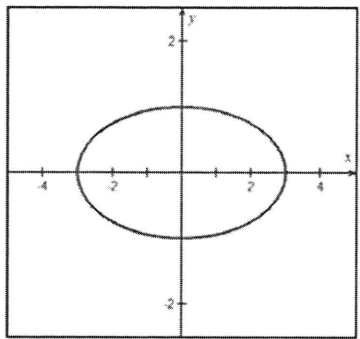
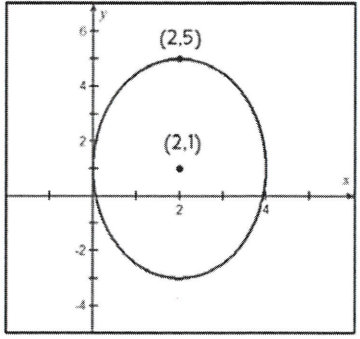
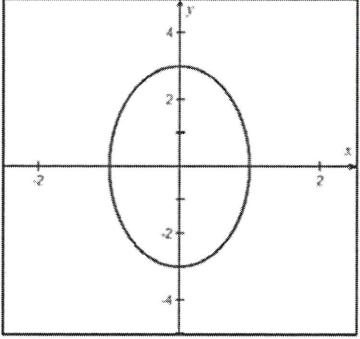
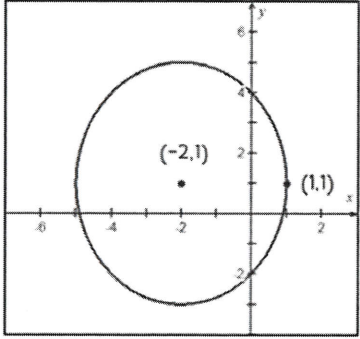
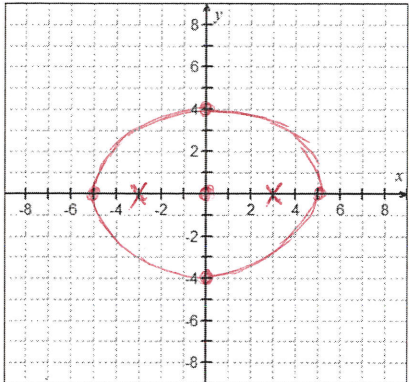
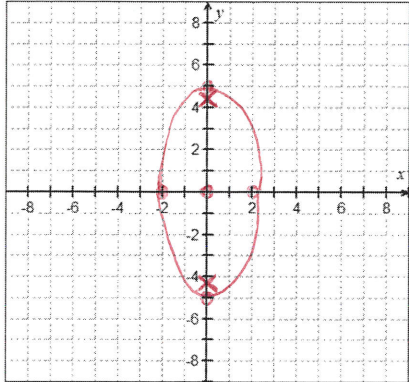


Ellipses Homework

Problems 1-6, the graph of an ellipse is given. Match each graph to its equation.

A. $\frac{x^2}{9} + y^2 = 1$ 3	B. $x^2 + \frac{y^2}{9} = 1$ 5	C. $\frac{x^2}{16} + \frac{y^2}{9} = 1$ 1
D. $\frac{(x+2)^2}{9} + \frac{(y-1)^2}{16} = 1$ 6	E. $\frac{(x-2)^2}{4} + \frac{(y-1)^2}{16} = 1$ 4	F. $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$ 2
1. C 	2. F 	3. A 
4. E 	5. B 	6. D 

Problems 7-12, find the center, vertices (major and minor), and foci of each ellipse, and write the equation if it is not already given. Graph each equation, clearly showing the major and minor axes, the foci, and the center.

7. $\frac{x^2}{25} + \frac{y^2}{16} = 1$ horizontal $a=5$ $b=4$ $c=3$ Center: $(0,0)$ MV: $(\pm 5, 0)$ F: $(\pm 3, 0)$ mV: $(0, \pm 4)$ 	8. $\frac{x^2}{4} + \frac{y^2}{25} = 1$ vertical $a=5$ $b=2$ $c=\sqrt{21}$ ≈ 4.6 Center: $(0,0)$ MV: $(0, \pm 5)$ F: $(0, \pm \sqrt{21})$ mV: $(\pm 2, 0)$ 
---	---

9. $4x^2 + 9y^2 = 36$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

Center: $(0,0)$

MV: $(\pm 3,0)$

mV: $(0, \pm 2)$

f: $(\pm\sqrt{5}, 0)$

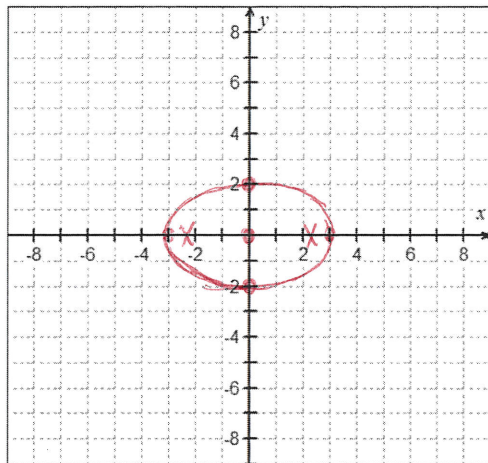
horizontal

$a=3$

$b=2$

$c=\sqrt{5}$

≈ 2.2



10. Focus at $(4, 0)$; vertices at $(\pm 5, 0)$

$c=4$

$a=5$

$4 = \sqrt{25 - b^2}$

$16 = 25 - b^2$

$b^2 = 9$

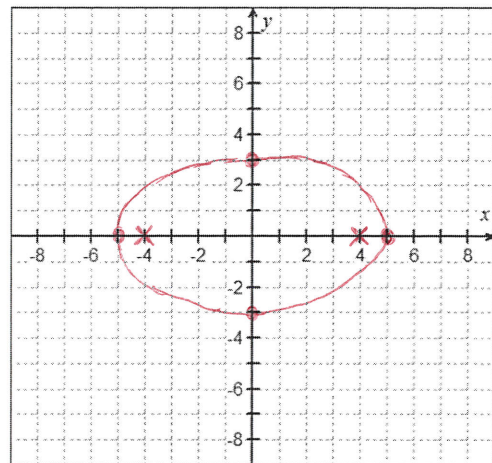
$b=3$

MV: $(\pm 5, 0)$

mV: $(0, \pm 3)$

f: $(\pm 4, 0)$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$



11. Foci at $(3, -4)$ and $(3, 0)$; major axis of length 12

Center: $(3, -2)$

$a=6$

MV: $(3, 4)$ $(3, -8)$

$c=2$

mV: $(3 \pm 4\sqrt{2}, -2)$

$2 = \sqrt{36 - b^2}$

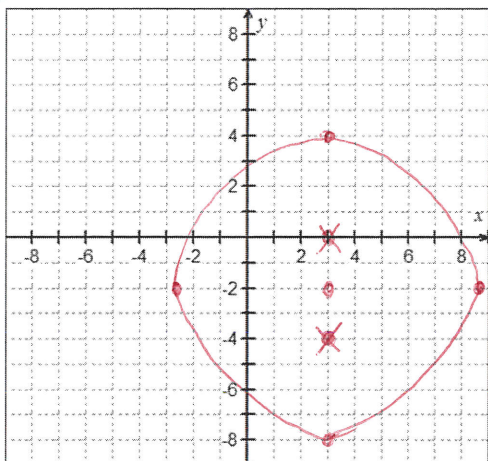
$4 = 36 - b^2$

f: $(3, 0)$ $(3, -4)$

$b^2 = 32$

$b = 4\sqrt{2} \approx 5.7$

$$\frac{(x-3)^2}{32} + \frac{(y+2)^2}{36} = 1$$



12. $3x^2 + y^2 = 12$

$$\frac{x^2}{4} + \frac{y^2}{12} = 1$$

vertical

$a = 2\sqrt{3} \approx 3.5$

$b=2$

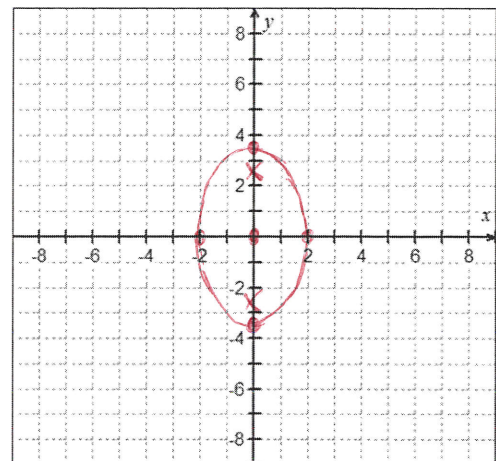
$c = 2\sqrt{2} \approx 2.8$

center: $(0,0)$

MV: $(0, \pm 2\sqrt{3})$

mV: $(\pm 2, 0)$

f: $(0, \pm 2\sqrt{2})$



Problems 13 – 16, find the center, foci, and vertices of each ellipse. Graph each equation, clearly showing the major and minor axes, the foci, and the center.

13. $\frac{(x-2)^2}{9} + \frac{(y+3)^2}{16} = 1$

vertical

center: (2, -3)

a=4

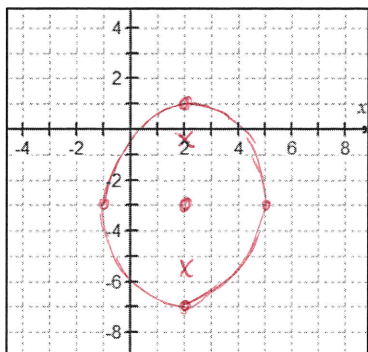
MV: (2, 1) (2, -7)

b=3

mV: (-1, -3) (5, -3)

c = $\sqrt{7} \approx 2.6$

f: (2, -3 ± $\sqrt{7}$)



14. $\frac{(x+4)^2}{16} + \frac{(y+2)^2}{25} = 1$

vertical

center: (-4, -2)

a=5

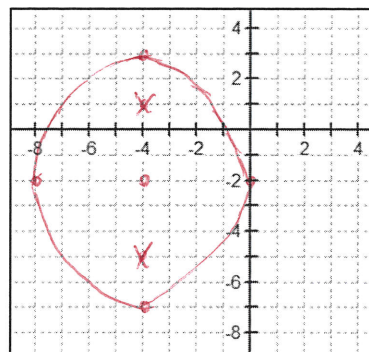
MV: (-4, 3) (-4, -7)

b=4

mV: (-8, -2) (0, -2)

c=3

foci: (-4, 1) (-4, -5)



15. $x^2 + 4x + 4y^2 - 16y - 16 = 0$

$x^2 + 4x + 4 + 4(y^2 - 4y + 4) = 16 + 4 + 16$

$(x+2)^2 + 4(y-2)^2 = 36$

$\frac{(x+2)^2}{36} + \frac{(y-2)^2}{9} = 1$

horizontal

a=6

b=3

center: (-2, 2)

c = $3\sqrt{3} \approx 5.2$

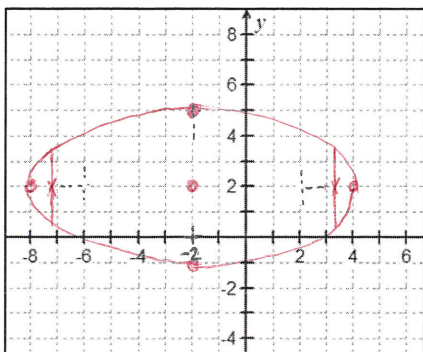
MV: (4, 2) (-8, 2)

mV: (2, 5) (2, -1)

latera recta:

$\frac{2b^2}{a} = \frac{18}{6} = 3$

Foci: (-2 ± $3\sqrt{3}$)



16. $4x^2 + 25y^2 - 24x - 100y + 36 = 0$

$4(x^2 - 6x + 9) + 25(y^2 - 4y + 4) = -36 + 36 + 100$

$4(x-3)^2 + 25(y-2)^2 = 100$

$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{4} = 1$

horizontal

a=5, b=2

c = $\sqrt{21} \approx 4.6$

center: (3, 2)

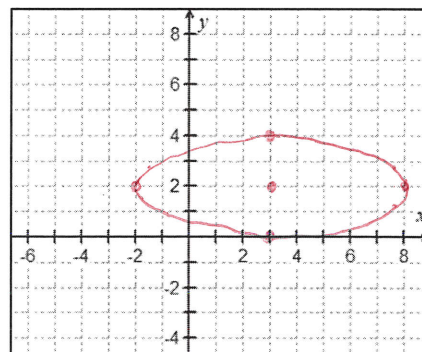
MV: (8, 2) (-2, 2)

mV: (3, 4) (3, 0)

latera recta:

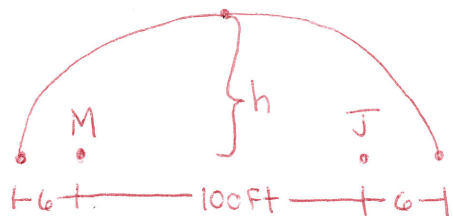
$\frac{2b^2}{a} = \frac{8}{5} \approx 1.6$

Foci: (3 ± $\sqrt{21}$, 2)



Problems 17–19, solve.

17. Madison is standing at one focus of a semi-elliptical whispering gallery, and is 6 feet away from the nearest wall. Her friend Julia is standing at the other focus, 100 feet away. What is the length of the whispering gallery? How high is its ceiling at the center?



$$\text{length} = \boxed{112 \text{ feet}}$$

$$a = 50, c = 50$$

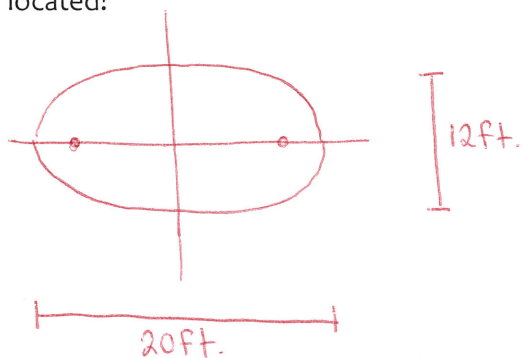
b will represent the height

$$c^2 = a^2 - b^2$$

$$50^2 = 50^2 - b^2$$

$$\boxed{b \approx 25.22 \text{ ft.}}$$

18. A reflecting pool in the shape of an ellipse is built in front of City Hall. The architect plans to place fountains at the foci. If the pool of water is 20 feet long and 12 feet wide, how far from the center should the fountains be located?



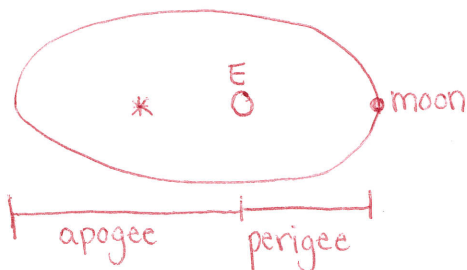
$$a = 10, b = 6$$

$$c^2 = 100 - 36$$

$$c = 8$$

$$\boxed{8 \text{ feet from the center}}$$

19. The moon travels around Earth in an elliptical orbit, with Earth at one focus. The major axis of the orbit is 768,800 kilometers and the minor axis is 767,640 kilometers. Find the apogee (greatest distance) and the perigee (smallest distance) between the center of the Earth and the center of the moon.



$$2a = 768,800$$

$$2b = 767,640$$

$$a = 384,400$$

$$b = 383,820$$

$$c^2 = a^2 - b^2 \quad \therefore c \approx 21,108$$

$$\text{apogee: } a + c \approx 405,508 \text{ km}$$

$$\text{perigee: } a - c \approx 363,292 \text{ km}$$